Coupled-mode theory of nonlinear propagation in multimode and single-mode fibers: envelope solitons and self-confinement

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A set of equations describing pulse propagation in multimode optical fibers in the presence of an intensity-dependent refractive index is derived by taking advantage of the coupled-mode theory usually employed for describing the influence of fiber imperfections on linear propagation. This approach takes into account in a natural way the role of the waveguide structure in terms of the propagation constants and the spatial configurations of the propagating modes and can be applied to the most general refractive-index distribution. The conditions under which soliton propagation and longitudinal self-confinement can be achieved are examined.

INTRODUCTION

The nonlinear response of a dielectric medium to electromagnetic radiation includes contributions from quadratic, cubic, and higher-order terms in the electric field. Some of the related nonlinear effects can be most conveniently observed at relatively low powers over the long interaction lengths provided by optical fibers. This circumstance can also make these processes detrimental for telecommunications fibers since they influence signal attenuation and dispersion. Among the effects associated with third-order nonlinearity, stimulated Raman scattering and stimulated Brillouin scattering limit the maximum input power available for transmission,[1] whereas self-phase modulation directly influences dispersion by modifying the pulse shape[2] and thus can play a relevant role, together with chromatic dispersion, in determining the transmission rate attainable in a given fiber.

Self-phase modulation is associated with the intensity-dependent self-induced changes of the refractive index resulting from the contribution to the third-order polarizability proportional to the field itself.[3] These variations of the refractive index, proportional to the instantaneous intensity of the field, give rise to self-focusing,[4] an effect negligible in fibers at the low powers usually employed, and to a phase modulation of the propagating pulse connected with the phase changes induced by the pulse itself. The frequency broadening associated with this nonlinear self-phase modulation tends either to narrow or to broaden the pulse according to whether the so-called group dispersion (that is, the second derivative with respect to ω of the propagation constant evaluated at the average frequency) is negative (anomalous dispersion) or positive (normal dispersion).[5] As was first pointed out by Hasegawa and Tappert,[6] it is possible, in the anomalous-dispersion regime, to take advantage of this effect exactly to balance the broadening that is due to chromatic dispersion by suitably choosing the intensity and shape of the pulse, thus achieving propagation of dispersionless pulses (bright-envelope solitons). The potentiality that the use of envelope transmission offers for obtaining very-high-transmission rates in actual fiber lines has been discussed.[7] In general, self-phase modulation is studied in single-mode fibers,[2] and the results of Ref. 6, as well as successive work putting into evidence the influence on soliton transmission of the radial dependence of the refractive index and of its eventual longitudinal dependence,[8,9] apply to this situation. It is obvious that the description of nonlinear propagation becomes more involved for a multimode fiber since modal dispersion, associated with the different group velocities of the various modes, now comes into play. Also, if it is to be expected that the possibility of propagating envelope solitons no longer exists, it is however possible to show that, under suitable conditions, the various modes interact among themselves in such a way as to give rise to a self-confinement mechanism that prevents the pulse from broadening as a consequence of modal dispersion.[10]

It has been shown that the coupled-mode formalism, usually employed for describing the process of energy exchange among the modes of a multimode waveguide resulting from the presence of imperfections,[11] turns out to provide a natural approach for studying nonlinear propagation[12,13] in single-mode and multimode optical fibers.

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In this paper, we present this method in full detail and examine the nonlinear evolution of a pulse propagating in a fiber supporting an arbitrary number of modes (for instance, the two polarization states of a single-mode fiber). This is accomplished by writing a general expression for the nonlinear dielectric-constant tensor, which in particular is specialized to the cases of polarization-maintaining and polarization-scrambling optical fibers. The pulse evolution is shown to obey a set of nonlinear coupled differential equations, whose structure is given also for the case of two counterpropagating modes. The problem of the existence of envelope-soliton solutions is then examined and the relative conditions stated. The hypotheses leading to the possibility of self-confinement are rigorously formulated, explicitly taking into account the influence of the spatial configurations of the various modes and of their mutual overlap. These theoretical investigations of self-phase modulation are encouraged by the fact that two experiments with single-mode fibers have been performed that successfully verify the predictions of pulse narrowing[14] and pulse broadening,[15] respectively, in the anomalous- and normal-dispersion regimes (that is, at wavelengths respectively longer or shorter than 1.3 μm for fused-silica fibers).

COUPLED-MODE-THEORY APPROACH TO NONLINEAR PROPAGATION

Let us briefly recall the formalism of coupled-mode theory developed for describing time-dependent propagation in the presence of fiber imperfections.[11,16] Denoting by $\mathbf{E}_\omega$ and $\mathbf{H}_\omega$ the time-Fourier transforms at the angular frequency ω of the electric and magnetic fields and by $\epsilon(x, y, z, \omega)$ the dielectric constant pertaining to the perturbed fiber, one has to find the solutions of Maxwell's equations

$$\begin{aligned}\nabla \times \mathbf{H}_\omega &= i\omega\epsilon\mathbf{E}_\omega,\\ \nabla \times \mathbf{E}_\omega &= -i\omega\mu_0\mathbf{H}_\omega,\end{aligned} \tag{1}$$

where μ_0 is the magnetic permeability of vacuum. To this end $\mathbf{E}_\omega$ and $\mathbf{H}_\omega$ are expressed as linear combinations of the modes $\mathbf{E}_n(\omega)\exp[-i\beta_n(\omega)z]$ and $\mathbf{H}_n(\omega)\exp[-i\beta_n(\omega)z]$ pertaining to the ideal waveguide characterized by a dielectric constant $\epsilon_1(x, y, \omega)$ with z-dependent expansion coefficients. Following this procedure (normal mode-expansion technique), one obtains a set of linear coupled differential equations for the expansion coefficients, which are equivalent to Maxwell's equations and generate solutions automatically satisfying the boundary conditions for the unperturbed fiber. More specifically, if one writes the forward-traveling transverse part of the electric field in the form

$$\mathbf{E}_T(\mathbf{r}, z, t) = \sum_m \int_{-\infty}^{+\infty} \mathbf{E}_{mT}(\mathbf{r}, \omega)c_m(z, \omega) \times \exp[i\omega t - i\beta_m(\omega)z]\mathrm{d}\omega, \tag{2}$$

where $\mathbf{r} = (x, y)$ indicates the transverse coordinates and

$$\mathbf{E}_m(\mathbf{r}, \omega) = \mathbf{E}_{mT}(\mathbf{r}, \omega) + \mathbf{E}_{mz}(\mathbf{r}, \omega), \tag{3}$$

the expansion coefficients c_m can be shown to obey the set of equations[11]

$$\frac{\mathrm{d}}{\mathrm{d}z} c_m(z, \omega) = \sum_n K_{mn}(z, \omega)\exp\{i[\beta_m(\omega) - \beta_n(\omega)]z\}c_n(z, \omega), \quad m = 1, 2, \ldots, \tag{4}$$

with

$$K_{mn} = (\omega/4iP) \iint_{-\infty}^{+\infty} (\epsilon - \epsilon_1)[\mathbf{E}_{mT}(\mathbf{r}, \omega) \cdot \mathbf{E}_{nT}(\mathbf{r}, \omega) + (\epsilon_1/\epsilon)\mathbf{E}_{mz}^*(\mathbf{r}, \omega) \cdot \mathbf{E}_{nz}(\mathbf{r}, \omega)]\mathrm{d}x\mathrm{d}y \tag{5}$$

and

$$P = (1/2) \iint_{-\infty}^{+\infty} \mathbf{e}_z \cdot \mathbf{E}_{mT}(\mathbf{r}, \omega) \times \mathbf{H}_{mT}(\mathbf{r}, \omega)\mathrm{d}x\mathrm{d}y, \tag{6}$$

$\mathbf{e}_z$ being a unit vector in the positive $\mathbf{z}$ direction. The normalization constant P does not depend on the mode, and the transverse parts of $\mathbf{E}_m$ and $\mathbf{H}_m$ are assumed to be real, the relative longitudinal parts then being purely imaginary quantities.

The above formalism can be applied to describe situations in which the departure of the fiber from ideal behavior is associated with the presence of the nonlinear intensity-dependent contribution to the dielectric constant. More precisely, by assuming the fiber material to be isotropic and the nonlinear response to be dominated by the fast-responding electronic processes, one can approximate the third-order nonlinear electric polarization by its dispersionless form[3]:

$$\mathbf{P}^{(3)} = \epsilon_0\chi^{(3)}\mathbf{E} \cdot \mathbf{E}\mathbf{E}, \tag{7}$$

where ϵ_0 is the electric permeability of vacuum and $\chi^{(3)}$ is the nonlinear susceptibility. After introduction of the analytic representation $\hat{\mathbf{E}}$ of the electric field satisfying the relation $\mathbf{E} = (\hat{\mathbf{E}} + \hat{\mathbf{E}}^*)/2$, the terms on the right-hand side of Eq. (7) that are vibrating at (approximately) the positive frequency ω are given by

$$\mathbf{P}_\omega{}^{(3)} = \epsilon^{(3)} : \mathbf{E}_\omega, \tag{8}$$

where the nonlinear tensor $\epsilon^{(3)}$ can be expressed by the matrix

$$\epsilon^{(3)} = \epsilon_0\chi^{(3)}/2 \begin{Vmatrix} \hat{\mathbf{E}} \cdot \hat{\mathbf{E}}^* + 1/2|\hat{E}_x|^2 & 1/2\hat{E}_y\hat{E}_x^* & 1/2\hat{E}_z\hat{E}_x^* \\ 1/2\hat{E}_x\hat{E}_y^* & \hat{\mathbf{E}} \cdot \hat{\mathbf{E}}^* + 1/2|\hat{E}_y|^2 & 1/2\hat{E}_z\hat{E}_y^* \\ 1/2\hat{E}_x\hat{E}_z^* & 1/2\hat{E}_y\hat{E}_z^* & \hat{\mathbf{E}} \cdot \hat{\mathbf{E}}^* + 1/2|\hat{E}_z|^2 \end{Vmatrix}. \tag{9}$$

According to Eq. (8), the relation between the electric displacement vector $\mathbf{D}$ and $\mathbf{E}$ becomes

$$\begin{aligned}\mathbf{D}_\omega &= \epsilon_0\mathbf{E}_\omega + \mathbf{P}_\omega\\ &= \epsilon_1(\omega)\mathbf{E}_\omega + \epsilon^{(3)} : \mathbf{E}_\omega = \epsilon : \mathbf{E}_\omega,\end{aligned} \tag{10}$$

$\epsilon_1(\omega)$ being the linear dielectric constant of the medium, and thus it exhibits a tensorial character that does not allow for a straightforward application of the results of the coupled-mode theory, in which the dielectric constant has been assumed to be a scalar quantity [see Eqs. (1)]. This difficulty can be circumvented as follows.

In most practical situations, one employs weakly guiding structures for which the longitudinal part of the electric field can be neglected with respect to the transversal part ($E_z \ll E_T$). This allows us to neglect the terms containing $\hat{E}_z$ in the matrix given in Eq. (9) so that

$$\mathbf{D}_\omega = \epsilon_1(\omega)\mathbf{E} + \epsilon_T{}^{(3)} : \mathbf{E}_{T\omega} + \epsilon_z{}^{(3)}\mathbf{E}_{z\omega}, \tag{11}$$

with

$$\epsilon_T{}^{(3)} = \frac{\epsilon_0\chi^{(3)}}{2}\begin{vmatrix} \hat{\mathbf{E}}\cdot\hat{\mathbf{E}}^* + \tfrac{1}{2}|\hat{E}_x|^2 & \tfrac{1}{2}\hat{E}_y\hat{E}_x{}^* \\ \tfrac{1}{2}\hat{E}_x\hat{E}_y{}^* & \hat{\mathbf{E}}\cdot\hat{\mathbf{E}}^* + \tfrac{1}{2}|\hat{E}_y|^2 \end{vmatrix}, \tag{12}$$

$$\epsilon_z{}^{(3)} = \tfrac{1}{2}\epsilon_0\chi^{(3)}\hat{\mathbf{E}}\cdot\hat{\mathbf{E}}^*, \tag{13}$$

and $\hat{\mathbf{E}}\cdot\hat{\mathbf{E}}^* = |\hat{E}_x|^2 + |\hat{E}_y|^2$. It is now possible to repeat the derivation leading from Eqs. (1) to Eq. (5) and show that the contribution arising from the last term on the right-hand side of Eq. (11) affects only the part of the coupling coefficients K_{mn} containing $E_{mz}{}^*E_{nz}$; since for weakly guiding fibers this term can be neglected, we can drop the last term in Eq. (11), and the problem needs to be treated only in the transverse plane x, y.

Furthermore, the matrix appearing in Eq. (12) can be put in diagonal form whenever $\hat{E}_y{}^*\hat{E}_x$ can be assumed to be zero. This is obviously the case for nonideal circularly symmetrical fibers in which the polarization of an initially linearly polarized field becomes random over short propagation lengths because of minor perturbations along the fiber so that, on the average, $|\hat{E}_x|^2 = |\hat{E}_y|^2$, $\hat{E}_y\hat{E}_x{}^* = 0$, and

$$\mathbf{D}_\omega = \epsilon\mathbf{E}_\omega, \tag{14}$$

where

$$\epsilon = \epsilon_1 + \epsilon_2|\hat{E}_x|^2, \tag{15}$$

with $\epsilon_2 = 5\epsilon_0\chi^{(3)}/4$. Other situations in which $\hat{E}_y\hat{E}_x{}^* = 0$ are those pertaining to a circularly symmetrical ideal fiber (no imperfections) and to a polarization-maintaining birefringent fiber (either single-mode[17] or multimode[18]) with principal axes $\hat{x}$ and $\hat{y}$, provided that the input field is linearly polarized along a generic direction in the first case and along $\hat{x}$ or $\hat{y}$ in the second one. In fact, the structure of $\epsilon_T{}^{(3)}$ in Eq. (12) implies that if $\hat{E}_y\,(\hat{E}_x)$ is initially zero it remains zero for every value of z. Thus Eqs. (14) and (15) still apply when $\epsilon_2 = 3\epsilon_0\chi^{(3)}/4$. Accordingly, we have a scalar dielectric constant and, correspondingly, a refractive index of the form

$$n = (\epsilon/\epsilon_0)^{1/2} \sim n_1 + n_2|\hat{E}_x|^2, \tag{16}$$

with

$$n_2 = \frac{1}{2n_1}\epsilon_2/\epsilon_0.$$

We can now apply the results of the coupled-mode formalism [Eqs. (1)–(6)] by writing ϵ in the form furnished by Eq. (15), provided that the characteristic time variations of the instantaneous intensity $|E_x|^2$ are slow compared with the period of the field, that is,

$$\frac{\delta\omega}{\omega_0} \ll 1, \tag{17}$$

where $\delta\omega$ is the bandwidth and ω_0 is the average frequency of the field. Usually the field is supposed to consist of a monochromatic carrier wave of frequency ω_0 whose amplitude modulation is entirely responsible for the bandwidth $\delta\omega$; an attempt to include the effects of the source fluctuations on nonlinear propagation has been made in Ref. 19. Proceeding in this way, we obtain the following set of nonlinear coupled equations describing the evolution of the mode amplitude *pertaining to a given state of polarization:*

$$\begin{aligned}\frac{\mathrm{d}c_m(z,\omega)}{\mathrm{d}z} &= -4i\sum_n c_n(z,\omega)\exp\{i\,[\beta_m(\omega)-\beta_n(\omega)]\}\sum_p\sum_q \\ &\times \iint_0^{+\infty} R_{pq}^{mn}(\omega,\omega',\omega'')c_p(z,\omega')c_q{}^*(z,\omega'') \\ &\times \exp[i(\omega'-\omega'')t]\exp\{-i[\beta_p(\omega')-\beta_q(\omega'')]\}\mathrm{d}\omega'\mathrm{d}\omega'', \\ &\qquad m = 1, 2, \ldots,\end{aligned} \tag{18}$$

where

$$R_{pq}^{mn}(\omega,\omega',\omega'') = \frac{\omega n_2}{c} \times \frac{\iint_{-\infty}^{+\infty} E_{mT}(\mathbf{r},\omega)E_{nT}(\mathbf{r},\omega)E_{pT}(\mathbf{r},\omega')E_{qT}(\mathbf{r},\omega'')\mathrm{d}x\mathrm{d}y}{\iint_{-\infty}^{+\infty} E_{mT}{}^2(\mathbf{r},\omega)\mathrm{d}x\mathrm{d}y}, \tag{19}$$

having taken advantage on the approximate relation

$$\mathbf{e}_z \times \mathbf{H}_{mT} = (\epsilon_0/\mu_0)^{1/2} n_1 \mathbf{E}_{mT} \tag{20}$$

valid for weakly guiding fibers.

If we now observe that $\mathbf{E}_m(\mathbf{r},\omega)$ are smoothly varying functions of ω, and if we assume, in full generality, that the field is the superposition of the various modes, each centered at its own frequency ω_m, then

$$\mathbf{E}_{mT} = \sum_m E_m(\mathbf{r},\omega_m)\hat{x} \times \int_{-\infty}^{+\infty} c_m(z,\omega)\exp\{i[\omega t - \beta_m(\omega)]z\}\mathrm{d}\omega, \tag{21}$$

where $\hat{x}$ is a unit vector in the direction of one of the principal axes if the fiber is birefringent or in an arbitrary direction otherwise, and Eqs. (18) become

$$\begin{aligned}\frac{\mathrm{d}c_m(z,\omega)}{\mathrm{d}z} &= -i\sum_n c_n(z,\omega)\exp\{i[\beta_m(\omega)-\beta_n(\omega)]z\} \\ &\times \sum_m\sum_n R_{pq}^{mn}(\omega_n,\omega_p,\omega_q)\hat{\psi}_p(z,t)\hat{\psi}_q{}^*(z,t), \\ &\qquad m = 1, 2, \ldots,\end{aligned} \tag{22}$$

where

$$\hat{\psi}_m(z,t) = 2\int_0^{+\infty} c_m(z,\omega)\exp[i\omega t - i\beta_m(\omega)z]\mathrm{d}\omega. \tag{23}$$

By multiplying both sides of Eq. (21) by $\exp[-i\beta_m(\omega)z + i\omega t]$ and integrating over ω in the interval $(0, +\infty)$, we obtain for the slowly varying amplitudes $\hat{\phi}_m(z,t)$ defined through the relation

$$\begin{aligned}\hat{\psi}_m &= \exp[i\omega_m t - i\beta_m(\omega)z]\hat{\phi}_m(z,t) \\ &= 2\exp[i\omega_m t - i\beta_m(\omega)z]\int_0^{+\infty} c_m(z,\omega) \\ &\quad \times \exp[i(\omega-\omega_m)(t - z/v_m) \\ &\quad - i(\omega-\omega_m)^2 z/2!A_m = i(\omega = \omega_m)^3/3!B_m]\mathrm{d}\omega\end{aligned} \tag{24}$$

a set of nonlinear coupled equations in the time domain that reads

$$L_m\hat{\phi}_m(z,t) = -i\sum_n\sum_p\sum_q R_{pq}^{mn}(\omega_n,\omega_p,\omega_q) \times \exp[i(\omega_n-\omega_m+\omega_p-\omega_q)t] \times \exp\{-i[\beta_n(\omega_n)-\beta_m(\omega_m)+\beta_p(\omega_p)-\beta_q(\omega_q)]z\} \times \hat{\phi}_n\hat{\phi}_p\hat{\phi}_q^*, \quad (25)$$

where we have introduced the differential operator

$$L_m = \frac{\partial}{\partial z} + \frac{1}{v_m}\frac{\partial}{\partial t} - \frac{i}{2A_m}\frac{\partial^2}{\partial t^2} - \frac{1}{3!B_m}\frac{\partial^3}{\partial t^3} + \ldots \quad (26)$$

and the group velocity v_m of the mth mode,

$$v_m = (\mathrm{d}\beta_m/\mathrm{d}\omega)^{-1}_{\omega=\omega_m}, \quad (27)$$

together with the group dispersion

$$A_m = (\mathrm{d}^2\beta_m/\mathrm{d}\omega^2)^{-1}|_{\omega=\omega_m} \quad (28)$$

and the second-order group dispersion

$$B_m = (\mathrm{d}^3\beta_m/\mathrm{d}\omega^3)^{-1}|_{\omega=\omega_m}. \quad (29)$$

Equation (25) can be substantially simplified if $\beta_m \neq \beta_n$ for all $m \neq n$. In fact, one can in this case neglect the terms on the right-hand side of Eq. (25) containing an exponential factor oscillating with z, thus obtaining

$$L_m\hat{\phi}_m = -i\hat{\phi}_m\left[\sum_n R_{nn}^{mm}(\omega_m,\omega_n,\omega_n)|\hat{\phi}_n|^2 + \sum_{n\neq m} R_{mn}^{mn}(\omega_n,\omega_m,\omega_n)|\hat{\phi}_n|^2\right], \qquad m = 1, 2, \ldots. \quad (30)$$

Equations (30) describe in full generality the longitudinal evolution of a pulse in a multimode optical fiber in the presence of an intensity-dependent dielectric constant whenever all the modes propagate in the same direction, but the calculations can easily be extended to include counterpropagating modes. We limit ourselves to reporting here the case of two counterpropagating modes of a single-mode fiber that is described by the two coupled equations

$$L_1{}^+\hat{\phi}_1{}^+ = -i\hat{\phi}_1{}^+R_{11}^{11}(|\hat{\phi}_1{}^+|^2 + 2|\hat{\phi}_1{}^-|^2),$$
$$L_1{}^-\hat{\phi}_1{}^- = i\hat{\phi}_1{}^-R_{11}^{11}(|\hat{\phi}_1{}^-|^2 + 2|\hat{\phi}_1{}^+|^2), \quad (31)$$

where $\hat{\phi}_1{}^+$ and $\hat{\phi}_1{}^-$ are the slowly varying amplitudes of the modes propagating, respectively, in the positive and negative z directions and

$$L_1{}^{(\pm)} = \frac{\partial}{\partial z} \pm \frac{1}{v_1}\frac{\partial}{\partial t} \mp \frac{i}{2!A_1}\frac{\partial^2}{\partial t^2} \mp \frac{1}{3!B_1}\frac{\partial^3}{\partial t^3}\ldots. \quad (32)$$

SOLITON PROPAGATION AND SELF-CONFINEMENT

The structure of the set of Eqs. (30) [or of Eqs. (31)] implies the constancy of the energy carried by each mode across the whole fiber section as a function of z, which is expressed by the relation

$$\frac{\mathrm{d}}{\mathrm{d}z}\int_{-\infty}^{+\infty} |\hat{\phi}_m(z,t)|^2\mathrm{d}t = 0. \quad (33)$$

Equation (33) can easily be proved by considering each equation of the set and its complex conjugate, by multiplying them, respectively, by $\hat{\phi}_m{}^*$ and $\hat{\phi}_m$, and by adding the resulting relations. In particular, according to Eq. (33), a mode initially not excited cannot gain energy from the other ones through the nonlinear coupling that we are considering.

Let us now consider the situation in which higher-order group dispersion can be neglected so that in the differential operators L_m it is possible to keep only the terms up to the second derivative in t.

Equations (30) can admit of particular solutions in the form of envelope solitons, provided that all the modes are made to travel with a common group velocity, that is, if their excitation frequencies ω_1 and ω_2 are chosen in such a way that

$$(\mathrm{d}\beta_1/\mathrm{d}\omega)_{\omega=\omega_1} = (\mathrm{d}\beta_2/\mathrm{d}\omega)_{\omega=\omega_2} = \ldots = 1/v. \quad (34)$$

Under this circumstance, it can be immediately verified that solutions of Eqs. (30) in the form of distortionless propagating pulses (so-called envelope solitons) are given by

$$\hat{\phi}_m(z,t) = \hat{\phi}_{om}\exp(iz/2A_m\tau^2)\mathrm{sech}[(t-z/v)/\tau], \qquad m = 1, 2, \ldots, \quad (35)$$

provided that the following relations between the temporal width τ and the amplitudes ϕ_{om} of the pulses

$$-\frac{1}{A_m\tau^2} = \sum_n R_{nn}^{mm}(\omega_m,\omega_n,\omega_n)|\hat{\phi}_{on}|^2 + \sum_{n\neq m} R_{mn}^{mn}(\omega_n,\omega_m,\omega_n)|\hat{\phi}_{on}|^2, \qquad m = 1, 2, \ldots \quad (36)$$

are satisfied. Since all the R_{mn}^{mn} are positive quantities, Eqs. (36) imply, as a necessary condition for the existence of solitons, that all the A_m are negative (anomalous *chromatic* dispersion), that is, that

$$\mathrm{d}^2\beta_m/\mathrm{d}\omega^2|_{\omega=\omega_m} < 0, \qquad m = 1, 2, \ldots. \quad (37)$$

Equations (36) furnish in a natural way the existence condition for soliton propagation in terms of the superposition integrals R_{mn}^{mn} and the propagation constants β_m of the various modes. In particular, the influence of the waveguide on the existence region of bright solitons (in its absence, bright-soliton propagation is possible only for anomalous *material* dispersion) is expressed by Eqs. (37), in which $\mathrm{d}^2\beta_m/\mathrm{d}\omega^2$ contains both material and waveguide dispersion, which is valid for any refractive-index profile.

In particular, in the case of a single-mode fiber, the soliton-existence condition takes the form

$$-\frac{1}{A_1\tau^2} = \frac{\omega_1}{c}n_2\alpha|\hat{\phi}_{01}|^2 \quad (38)$$

with

$$\alpha = \frac{\iint_{-\infty}^{+\infty} E_1{}^4(\mathbf{r})\mathrm{d}x\mathrm{d}y}{\iint_{-\infty}^{+\infty} E_1{}^2(\mathbf{r})\mathrm{d}x\mathrm{d}y}. \quad (39)$$

For single-mode fibers, $E_1(\mathbf{r})$ can be approximated by a Gaussian law,

$$E(\mathbf{r}) = \exp(-\tfrac{1}{2}r^2/r_0{}^2), \quad (40)$$

the spot size r_0 being a function of the refractive-index profile $n_1(\mathbf{r},\omega)$ and of the fiber-normalized frequency.[20] By inserting Eq. (40) into Eq. (39), we obtain $\alpha = \tfrac{1}{2}\pi$, so that Eq. (38) can be rewritten as

$$|\hat{\phi}_{01}|^2 = -\frac{2c}{\tau^2\omega_1 n_2}(\mathrm{d}^2\beta_1/\mathrm{d}\omega^2)_{\omega=\omega_1}$$
$$\simeq -\frac{2}{n_2}\frac{\lambda_1^2}{\omega_1^2\tau^2}(\mathrm{d}^2 n_1/\mathrm{d}\lambda^2)_{\lambda=\lambda_1} \quad (41)$$

[the last equality following from the assumption, valid in many practical cases, that $\mathrm{d}^2\beta_m/\mathrm{d}\omega^2 \simeq \mathrm{d}^2(\omega n_1/c)/\mathrm{d}\omega^2$], which coincides with the expression given in Ref. 7 apart from a factor π.

In general, Eqs. (30), with L_m truncated to include the term containing the second derivative with respect to time, do not admit of soliton solutions if, as usually happens, all the modes have a common excitation frequency and consequently different group velocities. In other words, the effect of the nonlinearity is not sufficient to balance modal dispersion exactly in a multimode fiber, as it does for chromatic dispersion in a monomode fiber. It has been pointed out,[10] however, that, in the case of anomalous dispersion, the nonlinear interaction among the various modes is capable, under suitable conditions for the mode amplitudes, of giving rise to a mutual attraction that prevents the modes from spreading too much, giving rise to a self-confinement mechanism.

In order to understand this mechanism, let us consider Eqs. (30) specialized to the case in which all the ω_n coincide with the central frequency of the carrier ω_0. The result is that

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_m}\frac{\partial}{\partial t}-\frac{i}{2A_m}\frac{\partial^2}{\partial t^2}\right)\hat{\phi}_m = -2i\hat{\phi}_m\left(\sum_{n\neq m} R_{nm}|\hat{\phi}_n|^2 + \tfrac{1}{2}R_{mm}|\hat{\phi}_m|^2\right), \quad (42)$$

where we have taken advantage of the fact that

$$R_{nn}^{mm}(\omega_0,\omega_0,\omega_0) = R_{mn}^{mn}(\omega_0,\omega_0,\omega_0) = R_{mn}. \quad (43)$$

By recalling Eq. (33), we can set

$$\int_{-\infty}^{+\infty} |\hat{\phi}_m(z,t)|^2 \mathrm{d}t = p_m \quad (44)$$

and define

$$\bar{t}_m(z) = \frac{1}{p_m}\int_{-\infty}^{+\infty} t|\hat{\phi}_m(z,t)|^2\mathrm{d}t, \quad (45)$$

which represents the average time of arrival, at a given position z, of the power carried by the mth mode. With a procedure completely analogous to that leading to Ehrenfest's theorem,[21] it is possible to show, starting from Eq. (42), that

$$\frac{\mathrm{d}}{\mathrm{d}z}\bar{t}_m = \frac{1}{v_m} - \frac{1}{p_m A_m}\int_{-\infty}^{+\infty}\hat{\phi}_m^*\frac{\partial}{\partial t}\hat{\phi}_m\mathrm{d}t \quad (46)$$

and

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{t}_m = -\frac{2}{p_m A_m}\sum_{n\neq m} R_{nm}\int_{-\infty}^{+\infty}|\hat{\phi}_m|^2\frac{\partial}{\partial t}|\hat{\phi}_n|^2\mathrm{d}t. \quad (47)$$

Furthermore, whenever a single mode is present,

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{t}_m = 0, \quad (48)$$

so that $\mathrm{d}\bar{t}_m/\mathrm{d}z$ does not depend on z (note in particular that for a soliton solution it coincides, as is to be expected, with $1/v_m$). For a multimode fiber, the $\bar{t}_m$ of a mode is influenced by the presence of the other modes in a way that lends itself naturally to a mechanical analogy, provided that the roles of z and t are interchanged. In fact, after introduction of the potential (which is attractive for $A_m < 0$),

$$V^{(m)}(z,t) = \frac{2}{p_m A_m}\sum_{n\neq m} R_{nm}|\hat{\phi}_n(z,t)|^2, \quad (49)$$

and, by supposing that $|\hat{\phi}_m(z,t)|^2$ varies, for every fixed z, on a time interval short compared with the typical variation time of $V^{(m)}(z,t)$, one can rewrite Eq. (47) as

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{t}_m = -\frac{\partial}{\partial t}V^{(m)}(z,t)|_{t=\bar{t}_m}. \quad (50)$$

If we now define

$$\bar{t} = \sum_m A_m p_m \bar{t}_m \Big/ \sum_m A_m p_m$$
$$\simeq \sum_m p_m\bar{t}_m/\sum_m p_m, \quad (51)$$

which represents the center of mass of the packet traveling in the fiber and resulting from the superposition of the various modes, it is possible to show, with the help of Eq. (47), that

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{t} = 0. \quad (52)$$

This fact allows us to rewrite Eq. (50) in the form

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\tau_m = -\frac{\mathrm{d}}{\mathrm{d}\tau_m}V^{(m)}(\tau_m), \quad (53)$$

where $\tau_m = \bar{t}_m - \bar{t}$ and $V^{(m)}(\tau_m) = V^{(m)}(z, \tau_m + \bar{t})$ can be considered approximately independent of z in the center-of-mass reference system to which Eq. (53) refers. By identifying Eq. (53) with the equation of motion of the center of mass of the mth mode, the condition for its trapping inside the packet can be immediately obtained by imposing the condition that its initial velocity be smaller than the escape velocity of the potential $V^{(m)}$, that is, that

$$\tfrac{1}{2}(1/v_m - 1/v_0)^2 \leq -\frac{2}{p_m A_m}\sum_{n\neq m} R_{nm}|\hat{\phi}_n(0,0)|^2, \quad (54)$$

where

$$\frac{\mathrm{d}}{\mathrm{d}z}\bar{t}_m|_{z=0} \simeq \frac{1}{v_m}, \quad (55)$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\bar{t}|_{z=0} \equiv \frac{1}{v_0}, \quad (56)$$

and we have assumed that the amplitude of each mode is centered approximately around $t = 0$ at $z = 0$. Equation (52) represents a more sophisticated version of the corresponding one worked out in Ref. 10 as it takes into account the relative weights of the various modes.

CONCLUSIONS

We have examined the effects of the intensity-dependent contribution to the refractive index on pulse propagation in a multimode optical fiber. By relying on the coupled-mode theory, we have developed a formalism that allows us to describe the evolution of the amplitudes of the various modes propagating along the fiber by means of a coupled system of nonlinear differential equations whose coefficients contain the overlap integrals of the mode spatial configurations. The formalism has then been employed to establish the existence condition for soliton propagation in multimode fibers and to recover the hypotheses under which longitudinal self-confinement can give rise, in competition with modal dispersion, to a compression mechanism that prevents pulse spreading.

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